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SPECTROSCOPY AND DECAYS OF HEAVY MESONS *

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ABSTRACT

Assuming Coulomb-like as well as confining scalar potential, we have solved Shrödinger equation perturbatively in $1/m_Q$ with a heavy quark mass m_Q . The lowest order equation is examined carefully. Mass levels are fitted with experimental data for D/B mesons at each level of perturbation. Meson wave functions obtained thereby can be used to calculate ordinary form factors as well as Isgur-Wise functions for semileptonic weak decays and other physical quantities. All the above calculations are expanded in $1/m_Q$ order by order to determine parameters as well as to compare with results of Heavy Quark Effective Theory.

1. Introduction

HQET (Heavy Quark Effective Theory),¹ is applied to many aspects of high energy theories and many kinds of physical quantities of QCD which can be perturbatively calculated in $1/m_Q$. Especially those regarding to B meson physics, e.g., the Isgur-Wise function of semileptonic weak decay processes $B \rightarrow D\ell\nu$ and the Kobayashi-Maskawa matrix element V_{cb} , have been calculated by many people. However, applications of HQET to higher order perturbative calculations are limited only to obtain forms of higher order operators, and their coefficients should be obtained so that results be fitted with experimental data. This is because most of the calculations based on HQET do not introduce heavy meson wave functions and hence there is no way to determine those coefficients within the model.

In this paper, we would like to start from introducing phenomenological dynamics, i.e., assuming Coulomb-like vector and confining scalar potential to $Q\bar{q}$ bound states (heavy mesons), expand a hamiltonian in $1/m_Q$ then perturbatively solve Shrödinger equation in $1/m_Q$. Angular part of the lowest order wave function is exactly solved. After extracting asymptotic forms of the lowest order wave function at both $r \rightarrow 0$ and $r \rightarrow \infty$ and adopting the variational method, we numerically obtain radial part of the trial wave function which is expanded in polynomials of radial variable r . Then fitting the smallest eigenvalues of a hamiltonian with masses of D and D^*

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mesons, a strong coupling α_s and other potential parameters are determined uniquely. Using parameters obtained this way, other mass levels are calculated and compared with experimental data for D/B mesons at each level of perturbation. Meson wave functions obtained thereby may be used to calculate ordinary form factors/Isgur-Wise functions for semileptonic weak decay processes and other physical quantities as well.

2. Hamiltonian and Eigenvalue Equation

Our hamiltonian is given by ²

$$H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q S(r) + \left\{ 1 - \frac{1}{2} [\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})] \right\} V(r).$$

When a heavy quark, Q , is treated as a non-relativistic particle, the hamiltonian is reduced into a 4×2 matrix operator and the Schrödinger equation becomes

$$(H_{-1} + H_0 + H_1 + H_2 + \dots) \phi = M \phi = (m_Q + E_b) \phi$$

where $m_q - E_b$ is a binding energy and scalar and vector potentials are given by

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4\alpha_s}{3r}, \quad \text{and} \quad E_b = E_0 + \left(\frac{m_q}{m_Q} \right) E_1 + \dots$$

and

$$\begin{aligned} H_{-1} &= m_Q \\ H_0 &= \vec{\alpha}_q \cdot \vec{p} + \beta_q (m_q + S) + V. \end{aligned}$$

Higher order terms like H_1 and H_2 have some complicated structures and are not described here for simplicity. Subscripts i of H_i denote the order of $1/m_Q$.

The -1st order eigenvalue equation is given by $H_{-1} \phi_0 = m_Q \phi_0$, which is a trivial one. The next 0-th order becomes a non-trivial equation, $H_0 \Psi_{jm}^k = E_k^0 \Psi_{jm}^k$, which can be reduced into

$$\begin{pmatrix} m_q + S + V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - S + V \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = E_k^0 \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}.$$

This equation is numerically solved by taking into account the asymptotic behaviors at both $r \rightarrow 0$ and $r \rightarrow \infty$ and their forms are given by

$$u_k(r), v_k(r) \sim w(r) r^\gamma \exp \left(-(m_q + b)r - \frac{1}{2} \left(\frac{r}{a} \right)^2 \right),$$

where $\gamma = \sqrt{k^2 - (4\alpha_s/3)^2}$ and $w(r)$ is some polynomial of r . In the above an eigenfunction, Ψ_{jm}^k , can, in general, be given by

$$\Psi_{jm}^k(\vec{r}, \Omega) = \Psi_k(r) y_{jm}^k(\Omega) = \frac{1}{r} \begin{pmatrix} u_k(r) \\ -iv_k(r) (\vec{\sigma}_q \cdot \vec{n}) \end{pmatrix} y_{jm}^k(\Omega),$$

where the function $y_{jm}^k(\Omega)$ is a linear combination of Y_{jm} and vector spherical harmonics $\vec{\sigma} \cdot \vec{X}_{jj}^m(\Omega)$ and $\vec{\sigma} \cdot \vec{Y}_{j(\pm)}^m(\Omega)$, which satisfies

$$(\vec{\sigma}_q \cdot \vec{\ell}) y_{jm}^k(\Omega) = -(k+1) y_{jm}^k(\Omega)$$

with $k = \pm j$, or $\pm(j+1)$ and $\vec{\ell}$ is an angular momentum. The first few series of y_{jm}^k functions are given by

$$y_{00}^1 = -\frac{1}{\sqrt{4\pi}} (\vec{n} \cdot \vec{\sigma}), \quad y_{00}^{-1} = \frac{1}{\sqrt{4\pi}},$$

$$y_{10}^1 = \frac{-i}{\sqrt{4\pi}} (\vec{n} \cdot \vec{\sigma}) \sigma_z, \quad y_{11}^1 = \frac{-i}{\sqrt{2\pi}} (\vec{n} \cdot \vec{\sigma}) \sigma_+, \quad y_{1-1}^1 = \frac{-i}{\sqrt{2\pi}} (\vec{n} \cdot \vec{\sigma}) \sigma_-,$$

where $y_{jm}^{-1} = -(\vec{n} \cdot \vec{\sigma}) y_{jm}^1$, $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$, and $\vec{n} = \vec{r}/r$.

Matrix elements of interaction terms among eigenfunctions are expressed as below. Degeneracy can be resolved by diagonal elements of H_1 and H_2 with respect to the k quantum number. Inclusion of off-diagonal elements of H_1 and H_2 are absorbed into wave function corrections. Calculating all the matrix elements from the hamiltonian given above, total mass matrix is given by,

$$\begin{pmatrix} E_{-1}+E_{-1,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{-1}+E_{-1,1} & 0 & 0 & 0 & 0 & V_{-1,2} & 0 \\ 0 & 0 & E_1+E_{1,0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_1+E_{1,1} & V_{1,-2} & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{-2,1} & E_{-2}+E_{-2,1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{-2}+E_{-2,2} & 0 & 0 \\ 0 & V_{2,-1} & 0 & 0 & 0 & 0 & E_2+E_{2,1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_2+E_{2,2} \end{pmatrix},$$

where

$$E_k = m_Q + E_k^{(0)} + E_k^{(1)} + E_k^{(2)}, \quad E_{k,j} = E_{k,j}^{(1)} + E_{k,j}^{(2)}, \quad V_{k,k'} = V_{k,k'}^{(1)} + V_{k,k'}^{(2)},$$

superscripts mean the order of $1/m_Q$, k and k' stand for k quantum number, and j for the total angular momentum.

Corrections to wavefunctions and eigenvalues can be calculated by applying the ordinary perturbation. For instance, in the above 8×8 mass matrix up to $O(1/m_c)$, the wave function for $(2, 2)$ element is given by

$$\begin{aligned} \Psi_2 &= \Psi_{1m}^{-1}(r, \Omega) - 7.92 \times 10^{-3} \Psi_{1m}^2(r, \Omega) \\ &= \Psi_{1m}^{-1}(r, \Omega) - 1.19 \times \left(\frac{m_u}{m_c}\right) \Psi_{1m}^2(r, \Omega), \end{aligned}$$

where

$$\alpha_s = \frac{g_s^2}{4\pi} = 0.299, \quad a = 2.10 \text{ GeV}^{-1}, \quad b = -0.084 \text{ GeV},$$

$$m_u = 0.01 \text{ GeV}, \quad m_c = 1.461 \text{ GeV}, \quad m_b = 4.92 \text{ GeV}.$$

If D and D^* are identified as Ψ_{00}^{-1} and Ψ_{1m}^{-1} , respectively, D and B meson masses are calculated to be

$$m_D = 1.867 \text{ GeV}, \quad m_{D^*} = 1.9241 \text{ GeV}, \quad m_B = 5.279 \text{ GeV}, \quad m_{B^*} = 5.298 \text{ GeV}.$$

3. Comments and Discussions

One can easily see degeneracy among the lowest lying pseudoscalar and vector states as follows. Define

$$|P\rangle = |D^\pm, D^0\rangle = \Psi_{00}^{-1}, \quad |V, m\rangle = |D^*\rangle = \Psi_{1m}^{-1},$$

where Ψ_{jm}^k is an eigenfunction obtained in the last chapter and quantum number k can take only $\pm j$, or $\pm(j+1)$. Since these states have the same quantum number $k = -1$, these have the same masses as well as the same wave functions up to the 0-th order calculation in $1/m_Q$. One needs to develop perturbation of energy and wave function for each state in terms of $1/m_Q$ to obtain higher order corrections.

Next we would like to discuss qualitative features of form factors functions. Let us think about to calculate form factors for semileptonic decay of $B \rightarrow D\ell\nu$. Taking a simple form for the lowest lying wave function both for B and D as

$$\Psi^{1S} \sim e^{-b^2 r^2/2},$$

where a parameter b is determined by a variational principle, $\delta(\Psi^{1S\dagger} H \Psi^{1S}) = 0$. Then form factors/Isgure-Wise functions are given by

$$F(q^2) \sim \exp \left[\text{const. } m_q^2 (q^2 - q_{\text{max}}^2) \right], \quad \text{or} \quad \xi(\omega) \sim \exp \left[\text{const. } m_q^2 (1 - \omega) \right],$$

where

$$q^2 = (p_B - p_D)^2, \quad \omega = v_B \cdot v_D, \quad q_{\text{max}}^2 = (m_B - m_D)^2 \leftrightarrow \omega_{\text{max}} = 1.$$

This means behavior of form factors strongly depends on a value of light quark mass m_q . One needs current quark mass for m_q to reproduce heavy meson mass spectrum, while constituent quark mass is apparently used to calculate form factors in all published papers. In order to explain this situation, one is forced to use running mass³ $m_q(q^2)$ for light quark mass.

4. References

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